Put $a=1$ and $b=2$. Then the roots of the four squares are 1, $\sqrt{2}$, $\sqrt{3}$, 2.

Put $a=1$ and $b=5$. Then the roots are 1, 3, $\sqrt{17}$, 5.

A similar proof was received from *CHARLES C. CROSS.*

II. **Solution by A. H. BELL, Hillsboro, Ill.**

Take $2y^2-x^2=\square$, or $x^2-2y^2=-\square=-1=-4$, etc.

In $x^2-2y^2=-1$......(3), the integral values for x and y are the alternate convergent fractions for the $\sqrt{2}$=to

$$x/y=1/1,\ 7/5,\ 41/29,\ \text{etc}......(4).$$

For the next, $x^2-2y^2=-4$. $(4)\times\sqrt{4}$,

$$\frac{x}{y}=\frac{1\times2}{1\times2},\quad \frac{7\times2}{5\times2},\quad \frac{41\times2}{29\times2},\quad \text{etc.}$$

Consequently the interchangeable values of x and y must be found in the first fraction and no other.

III. **Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.**

$2x^2-y^2=\square$......(1), and $2y^2-x^2=\square$......(2).

Take $x=my$ and $2m^2-1=\square$......(3), and $2-m^2=\square$......(4).

Then $m<\sqrt{2}$ and $m>\frac{1}{2}\sqrt{2}$.

It is manifest that both (3) and (4) are rational when $m=1$, which is $<\sqrt{2}$ and $>\frac{1}{2}\sqrt{2}$.

Then in (3) take $m=n+1$, and we have

$$2n^2+4n+1=\square=(\text{say})(qn-1)^2,\ \text{whence}$$

$$n=\frac{2(q+2)}{q^2-2}\ \text{and}\ m=n+1=\frac{(q+1)^2+1}{q^2-2}.$$

Substituting this value of m in (4) and reducing by the usual methods, we find $q=0$ and $m=\pm1$.

Hence $x=\pm y$ and the integral values are any equal numbers, positive or negative, or one positive and the other negative.

MISCELLANEOUS.

65. **Proposed by J. M. COLAW, A. M., Monterey, Va.**

Three circles, radii in ratio 1, 3, 5, are tangent externally and enclose one acre; what are the radii ?

Solution by A. H. BELL, Hillsboro, Ill.; J. SCHEFFER, A. M., Hagerstown, Md.; FREMONT CRANE, Sand Coulee, Mont., and COOPER D. SCHMITT, A. M. University of Tennessee, Knoxville, Tenn.

Let the radii be x, $3x$, and $5x$, respectively.

The sides of the triangle formed by joining the centers are $4x$, $6x$, and $8x$, and let the angles opposite each side, respectively, be A, B, and C; and then are found from the sides of the initial triangle 2, 3, and 4,

$$\cos A=\frac{9+16-4}{24}. \quad \therefore A=28^\circ\ 57'\ 18''\ \text{arc}=.5053601,$$

$$\cos B=\frac{4+16-9}{16}. \quad \therefore B=46^\circ\ 34'\ 3''\ \text{arc}=.8127562,$$

$$\cos C=\frac{4+9-16}{12}=-\tfrac{1}{4}. \quad \therefore C=104^\circ\ 28'\ 39''\ \text{arc}=1.8234763.$$

Now the triangle equals the three seetors=one acre=(10 square chains).

$$\text{Then}\ \frac{25x^2A}{2}+\frac{9x^2B}{2}+\frac{x^2C}{2}+10=24x^2\sin A \ldots\ldots\ldots (1).$$

$$\therefore x=\sqrt{\frac{20}{48\sin A-(25A+9B+C)}} \ldots\ldots\ldots (2).$$

$1-\cos^2 A=\sin^2 A$ and $\sin A=\frac{1}{8}\sqrt{15}=0.4841229$.

$x=3.694$ chains, and the radii are 3.694, 11.082, and 18.47 chains.

Also solved by *G. B. M. ZERR*, and the *PROPOSER*.

Solved by Elmer Schuyler, with results, 9.985 rods, 29.955 rods, and 49.925 rods; by Alois F. Kovarik with 19.903 rods, 57.709 rods, and 99.515 rods; and by Josiah H. Drummond with approximate results. The methods were all correct but there were some errors of calculation. Later a solution was received from Walter H. Drane, with results, 211.329+ feet, 633.987+ feet, and 1056.645+ feet.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

108. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A man who feels his death approach bequeathes to his young wife one-third of his fortune, and the remaining two-thirds to his son, if such should be born; but one-half of it to the widow and the other half to his daughter, if such should be born. After his death twins are born, a son and a daughter. How should the fortune be divided amongst the three ?

109. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Why do fences and telegraph poles appear to move rapidly in an opposite direction to one traveling in a railway car ? [From Moore's *Grammar School Arithmetic*, page 150.]

*** Solutions of these problems should be sent to B. F. Finkel not later than April 10.